



























`tsfilter hp` uses the Hodrick–Prescott (HP) high-pass filter to separate a time-series  $y_t$  into trend and cyclical components

$$y_t = \tau_t + c_t$$

where  $\tau_t$  is the trend component and  $c_t$  is the cyclical component.  $\tau_t$  may be nonstationary; it may contain a deterministic or a stochastic trend, as discussed below.

The primary objective is to estimate  $c_t$ , a stationary cyclical component that is driven by stochastic cycles at a range of periods. The trend component  $\tau_t$  is calculated by the difference  $\tau_t = y_t - c_t$ .

Formally, the filter is defined as the solution to the following optimization problem for  $\tau_t$

$$\min_{\tau_t} \left[ \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \{(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\}^2 \right]$$

where the smoothing parameter  $\lambda$  is set fixed to a value.

If  $\lambda = 0$ , the solution degenerates to  $\tau_t = y_t$ , in which case the filter excludes all frequencies, that is,  $c_t = 0$ . On the other extreme, as  $\lambda \rightarrow \infty$ , the solution approaches the least-squares fit to the line  $\tau_t = \beta_0 + \beta_1 t$ ; see Hodrick and Prescott (1997) for a discussion.

For a fixed  $\lambda$ , it can be shown that the cyclical component  $\mathbf{c}' = (c_1, c_2, \dots, c_T)$  is calculated by

$$\mathbf{c} = (\mathbf{I}_T - \mathbf{M}^{-1})\mathbf{y}$$

where  $\mathbf{y}$  is the column vector  $\mathbf{y}' = (y_1, y_2, \dots, y_T)$ ,  $\mathbf{I}_T$  is the  $T \times T$  identity matrix, and  $\mathbf{M}$  is the  $T \times T$  matrix:

$$\mathbf{M} = \begin{pmatrix} (1 + \lambda) & -2\lambda & \lambda & 0 & 0 & 0 & \dots & 0 \\ -2\lambda & (1 + 5\lambda) & -4\lambda & \lambda & 0 & 0 & \dots & 0 \\ \lambda & -4\lambda & (1 + 6\lambda) & -4\lambda & \lambda & 0 & \dots & 0 \\ 0 & \lambda & -4\lambda & (1 + 6\lambda) & -4\lambda & \lambda & \dots & 0 \\ \vdots & & & & & & \dots & \vdots \\ \vdots & 0 & & & & & & 0 \\ 0 & \dots & \lambda & -4\lambda & (1 + 6\lambda) & -4\lambda & \lambda & 0 \\ 0 & \dots & 0 & \lambda & -4\lambda & (1 + 6\lambda) & -4\lambda & \lambda \\ 0 & \dots & 0 & 0 & \lambda & -4\lambda & (1 + 5\lambda) & -2\lambda \\ 0 & \dots & 0 & 0 & 0 & \lambda & -2\lambda & (1 + \lambda) \end{pmatrix}$$

The gain of the HP filter is given by (see King and Rebelo [1993], Maravall and del Rio [2007], or Harvey and Trimbur [2008])

$$\psi(\omega) = \frac{4\lambda\{1 - \cos(\omega)\}^2}{1 + 4\lambda\{1 - \cos(\omega)\}^2}$$

As discussed in [TS] `tsfilter`, there are two approaches to selecting  $\lambda$ . One method, based on the heuristic argument of Hodrick and Prescott (1997), is used to compute the default values for  $\lambda$ . The method sets  $\lambda$  to 1,600 for quarterly data and to the rescaled values worked out by Ravn and Uhlig (2002). The rescaled default values for  $\lambda$  are 6.25 for yearly data, 100 for half-yearly data, 129,600 for monthly data,  $1600 \times 12^4$  for weekly data, and  $1600 \times (365/4)^4$  for daily data.

Additional technical description is available from Stata's [website](#).